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Metaphysics and Mathematics in Kant's *Critique*
a.k.a.
Kant's Unrequited Love for Mathematics[1]

Introduction

A running theme in Kant's (1996 [1787]) *Critique of Pure Reason* (hereafter simply *Critique* as I am dealing with this one only) is the relationship between mathematics and metaphysics. This paper will contain four sections dealing with an exploration of this theme. First, I include a section explaining what Kant took the goal and structure of metaphysics to be. Second, I discuss what Kant took mathematics to be about and its nature. Third, I include a section discussing his conception of the interrelation between the two. The fourth and final section contains a criticism of the Kantian accounts developed in the first three sections. This section will be something like the way a (pseudo) Leibnizian might want to answer some of the Kantian remarks.

Before I begin, I would like to make three notes concerning the nature of the present paper. First, I am using the Pulhar (1996) English translation of the *Critique* except where noted. (This edition nicely supplies some of the German and so is very useful for readers of some German that do not feel comfortable tackling Kant completely in German.) Second, I am considering the "B" edition of the *Critique* to be the text I am referring to except where noted. (References will still be provided in the standard A nnn / B mmm fashion.) Third, I regard this paper as a (to use Kant's own language) prolegomenon to some work[2] on the Kant-Newton connection. It thus has something of a "leading into" feel to it.

Section 1 - Kant's Metaphysics

Kant first uses the term "metaphysics" in the *Critique* at B xiv-xv where he writes (formatting added for reference - see below):

> "Metaphysics is a <u>speculative cognition by reason that is wholly isolated and rises entirely above being instructed by experience</u>. It is cognition through mere concepts (not, like mathematics, cognition through the application of concepts to intuition), so that here reason is to be its own pupil. But although metaphysics is older than all the other sciences, and **would endure even if all the others were to be engulfed utterly in the abyss of an all-annihilating barbarism**, fate thus far has not favored it to the point of enabling it to enter the secure path of a science. For in metaphysics reason continually falters, even when the laws into which it seeks to gain (as it <u>pretends) a priori insight</u> are those that are confirmed by the commonest experiences. Countless

[1] This subtitle is something of a joke, but I ask that the reader pay attention to section four of the paper, where its meaning will be explained.

[2] I am not intending to be a historian of either philosophy or science, so I doubt I will ever actually work on this. However, the subject is a direct continuation of the present work, as what one takes Kant's metaphysics and philosophy of mathematics to be affects what one takes to be his understanding of Newton. (To some degree, anyway.)

times, in metaphysics, we have to retrace our steps, because we find that our path does not lead us where we want to go. As regards agreement in the assertions made by its devotees, metaphysics is very far indeed from such agreement. It is, rather, a combat arena which seems to be destined quite specifically for practicing one's powers in mock combat, and in which not one fighter has ever been able to gain even the smallest territory and to base upon his victory a lasting possession. There can be no doubt, therefore, that the procedure of metaphysics has thus far been a mere groping about, and - worst of all - a groping about among mere concepts."

I extract three key points of Kant's conception of metaphysics from this passage. First (see the underlined phrase), he thinks that metaphysics is speculative and goes beyond experience. It appears to have no (direct?) use concerning the "phenomenal world" at all. Second (see the bold phrase), Kant thinks that metaphysics is basic to the human condition. Even if we lost all our other intellectual pursuits, we would still speculate in the domain of metaphysics. Finally (see the boxed phrase), Kant thinks that investigation of mere concepts alone does not suffice for metaphysics.

Let us look to the rest of the *Critique* where he develops each of these points in substantial detail. First, I will look at the transempirical nature of metaphysics. This is first thoroughly discussed at B 23 where Kant writes:

"[Here reason] deals with problems that issue entirely from its own womb; they are posed to it not by the nature of things distinct from it, but by its own nature. And thus, once it has become completely acquainted with its own ability regarding the objects that it may encounter in experience, reason must find it easy to determine, completely and safely, the range and the bounds of its use [when] attempted beyond all bounds of experience."

This paragraph tells us that the problems of metaphysics do not even have their origin in experience. They arise simply by the nature of humanity, which is the second of the three points (about Kantian metaphysics) above.

Another way (found elsewhere) this is expressed is when Kant is talking about the division between the branches of philosophy. At A 840 / B 868, we find:

"All philosophy, however, is either cognition from pure reason or rational cognition from empirical principles. The first is called pure and the second empirical philosophy."

To understand that this is referring to metaphysics, we have to find what Kant says pure philosophy and pure reason are. Kant thinks that pure reason is reason independent of all experience. So metaphysics does not originate in experience and does not refer to it. Let us look at more on the claim that it arises from human nature (the human condition). A clear exploration of this issue is found in the section concerning the "architectonic of pure reason", A 842 / B 870 where Kant explains this historically:

"What chemists do in separating kinds of matter and what mathematicians do in their pure doctrine of magnitudes is far more incumbent still on the philosopher, in order that he can reliably determine the share that a particular kind of cognition has in the roaming use of the understanding, i.e., determine its own value and influence. Hence human reason, ever since it has been thinking or-rather-meditating, has never been able to dispense with a metaphysics, yet has nonetheless been unable to expound one that was sufficiently purified of everything extraneous."

Metaphysics is thus regarded as a sort of higher order thinking process; almost a thinking about thinking. Because the ability to think includes the ability to think about thinking[3], and human beings are necessarily creatures which can think, it then follows that humans have always been concerned with metaphysics.

This historical look at the origin of metaphysics also tells us Kant's reason for why he thinks that even if all other intellectual pursuits were to be exterminated from human thoughts, humans would still be inclined to metaphysical speculation. As we have seen, Kant thinks that the ability to think necessitates the ability for self-reflective thinking. Since this is in some form what metaphysics is about, metaphysics is hence always possible and does not require any other discipline to ground it/support it/provide information for it.

Before we move on to what Kant expects metaphysics to be concerned with, we must finish this introduction on how metaphysics is characterized by a look at what Kant means by that metaphysics goes beyond groping around with "mere concepts".

Kant writes at B 23-24 most of the answer to this question and also gives us some material for understanding the task of metaphysics, which will be the next subject for me to discuss. He writes (italics in original):

> "Hence all attempts that have been made thus far to bring a metaphysics about *dogmatically* can and must be regarded as if they had never occurred. For whatever is analytic in one metaphysics or another, i.e. is mere dissection of the concepts residing a priori in our reason, is only a prearrangement for metaphysics proper and not yet its purpose at all. That purpose is to expand our a priori cognition synthetically, and for this purpose the dissection of a priori concepts is useless. For it shows merely what is contained in those concepts; it does not show how we arrive at such concepts a priori, so that we could then also determine the valid use of such concepts in regard to the objects of all cognition generally."

This explains the "going beyond groping" somewhat negatively. It explains why doing this groping is not productive. Kant tells us that because analysis of concepts only tells us what we already knew and not either the origin or the usefulness of these concepts, it (analysis) cannot be what metaphysics is about.

So, what then is Kant's conception of the domain of metaphysics? What sort of things does it deal with? Much later in the *Critique* (A 846-847 / B

[3] As we shall see in the section (four) of this paper on objections, this premise is a bit dubious. Nevertheless, it is what Kant seems to be saying here.

874-875) he divides metaphysics into four broad species, which are as follows: ontology, rational physiology, rational cosmology and rational theology. Rational physiology is in turn further divided into rational psychology and rational physics. He makes the claim that the idea of metaphysics itself forces this division upon us. He then proceeds to explain each of these in turn.

Ontology (the general study of objects) is said to regulate the possibility of experience. We are said to (A 848 / B 876) (italics in original):

> “[...] take from experience nothing more than what is needed to *give* us an object either of outer or of inner sense. The object of outer sense is given through the mere concept of matter (impenetrable, inanimate extension); the object of inner sense is given through the concept of a thinking being (in the empirical inner presentation *I think*).”

Kant writes that without these two basic ontological concepts (the self and the idea of matter) nothing we experience would be capable of making any sense to us.

Rational psychology, rational physics and rational cosmology are three branches of ontology which make the “empirical versions” of the fields with the same names possible. Rational psychology is presupposed by empirical psychology and rational physics is presupposed by empirical physics[4]. Rational cosmology works a bit different, because it also requires avoiding the first, second, and fourth antimonies of pure reason, whose significance for cosmological investigation is at A 408 / B 435: “System of Cosmological Ideas”.

Finally, rational theology is the branch of metaphysics which deals with a supreme being. Kant claims (see A 814-815 / B 842-843) that such a branch of metaphysics is required to complete what he calls transcendental and natural theology. This completion is necessary for Kant, as it allows for a moral unity in the world, and by extension of this, a unity generally. Thus rational theology is necessary for understanding the world at all[5].

Now that we have seen the nature and scope of Kantian metaphysics, let us now look at his philosophy of mathematics.

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[4] It appears that Kant here is under the misapprehension that Newtonian mechanics (or for that matter, even Aristotelian mechanics (!)) is strictly empirical. Kant is correct to point out that physics (and science generally) does require a metaphysics. It just so happens that it requires one of a radically different sort from the one he envisages. But this is another story for another time.

[5] One undercurrent that sweeps throughout the entire *Critique* is the collapsing of metaphysics and epistemology together. We shall see more of this in the section on Kant’s philosophy of mathematics, and some possible responses to this in the section on (pseudoLeibnizian) criticism of the *Critique*.

Section 2 - Kant’s Philosophy of Mathematics

“Mathematics” is first found in the *Critique* at B x, where Kant introduces sciences involving theoretical cognitions by reason. He writes (italics and brackets in original):

> “Two [sciences involving] theoretical cognitions by reason are to determine their *objects* a priori: they are *mathematics* and *physics*. In mathematics this determination is to be entirely pure; in physics it is to be at least partly pure, but to some extent also in accordance with sources of cognition other than reason.”

There are two important facts to be gleaned here about the Kantian conception of mathematics. First, that the objects of mathematics are said to be determined prior to experience. Second, mathematics is a theoretical science. The important question this raises is how mathematics is possible - how can the objects of investigation be determined prior to experience. Kant explains later that there are two possibilities (B 14-16). One is that mathematics is solely analytic. In other words, that it simply discovers new truths by examining the concepts that make up a concept which is already given. Kant rejects this at B 15 by saying that if one considers the proposition ‘7 + 5 = 12’, the concepts of ‘7’, ‘5’ and ‘+’ do not contain within them the concept of ‘12’. Hence arithmetic is not analytic. He proceeds to give another example from geometry, explaining that some propositions that are thought to be principles and analytic are not really principles (he gives as example that (a+b) > a, which he says expresses “the whole is greater than the part”). Having (he thinks) dealt with all of mathematics[6], the question then becomes how one can “build ideas” a priori.

Kant’s remarks on geometry and this problem are generally speaking clearer than the ones on other branches of mathematics, so let us start by examining the building of ideas in geometry. A 165-166 / B 206-207 explains the transcendental nature of mathematics, and geometry in particular (bold, underlining, boxing and inversion added:

> “This transcendental principle of the mathematics of appearances greatly expands our a priori cognition. For it alone is what makes pure mathematics in all its precision applicable to objects of experience. In the absence of the principle, this applicability might not be so self-evident, and has in fact been contested by many. For appearances are not things in themselves. **Empirical intuition is possible only through pure intuition (of space and time). Hence what geometry says about pure intuition holds incontestably for empirical intuition also**. And the subterfuges whereby objects of the senses need not conform to the rules of construction in space (e.g., the rule of the infinite divisibility of lines or angles) must be dropped. For by making them

[6] At Kant’s time, there were several other branches of mathematics besides geometry and arithmetic. These include: algebra (known in Europe since at least the 15th century) and tied to geometry in the 17th century by Descartes, the beginnings of analysis (Bernoullis, Leibniz, etc.), calculus (Newton and Leibniz), number theory, graph theory (Euler for the previous two) (see Dunham 1990). So he has perhaps jumped the gun a bit here - perhaps some other branches of mathematics are analytic. (However, his inductive generalization is in fact correct. That is, none of mathematics is analytic, but that is partially because an apparently better distinction that deals with how math fits into the system of human knowledge than analytic/synthetic is formal/factual.)

> one denies objective validity to space, and thereby also to all mathematics, and one no longer knows why and how mathematics is applicable to appearances. The synthesis of spaces and times, which are the essential form of all intuition, is what also makes possible the apprehension of appearance, hence makes possible any out experience, and consequently also makes possible all cognition of the objects of this experience. And thus what mathematics in its pure use proves for that synthesis holds necessarily also for this cognition. All objections against this are only the chicanery of a falsely instructed reason: a reason that erroneously means to detach objects of the senses from the formal condition of our sensibility, and despite their being mere appearances presents them as objects in themselves, given to the understanding. If that were the case, however, then there could be no synthetic a priori cognition of them at all, and hence also no such cognition through [redacted], [redacted], would itself not be possible."

This long passage tells of several key ideas concerning Kant's philosophy of geometry. First, (see the bold passages) Kant claims that due to our a priori conception of space, our a priori of intuition of the properties of space itself must therefore be applicable to empirical things. Thus geometry is able to be done a priori simply because phenomena must correspond to it. The world must correspond to the way we are set up to be. Second, (see the underlined passage) Kant thinks that all of mathematics deals with the idea of space, and not simply geometry. Third (see the boxed passage), geometry is said to be formal and tied to our sensibility. Fourth, (see the inverted passage) geometry is said to determine the pure concepts of space.

Each of these claims requires a little explanation. As we have seen, Kant thinks that we have an a priori conception of space, and in order for that conception to "work at all", it must be that space correspond to this intuition. But the only way in which Kant thinks this will work is if space itself is not (really) real but transcendentally ideal. Phenomena are "in space", but we cannot know the way things really are, as any cognition of things presupposes this intuition.

Kant's claim that all of mathematics involves the idea of space is rather obscure. I shall return to it when I discuss his conception of arithmetic, below. The claim that geometry is formal and tied to sensibility is simply the claim that it deals with the form of sensibility, and not what makes it up. Geometry deals with extension, space, distance, and so forth. All of these are not about the properties of (phenomenal) things, but about the form of our intuitions necessary for us to deal with the properties of (phenomenal) things. Finally, geometry's determining factor is important. Geometry's categories are the categories of space prior to all but necessary for all experience, as we have seen. Because geometry's categories are for Kant presupposed by all experience, geometry's subject matter cannot change, hence the determination. They are also determined by how they are linked by necessity[7], which Kant explains at B 41.

Kant's conception of arithmetic should be looked at in two parts. First,

[7] Kant seems to almost invent the notion of logical necessity here.

a look at how arithmetic differs from geometry, and second why Kant thinks that arithmetic involves the idea of space. This latter question was left dangling when it came up in the discussion of geometry.

For Kant, arithmetic differs from geometry in one important respect. It has no axioms. At A 164-165 / B 205-206 we find Kant's reasons for why arithmetic contains no axioms. He writes (italics in original, other formatting added):

> "The <u>evident propositions</u> of numerical relations, on the other hand, are indeed synthetic. <u>Yet, unlike those of geometry, they are not universal, and precisely because of this, they also cannot be called axioms, but can only be called numerical formulas</u>. The proposition that 7 + 5 = 12 is not an analytic proposition. For neither in the presentation of 7, nor in that of 5, nor in the presentation of the assembly of the two numbers do I think the number 12. (The fact that I ought to think the number 12 in *adding the two numbers* is not at issue here; for in analytic proposition the question is only whether I actually think the predicate in the presentation of the subject.) But although the proposition 7 + 5 = 12 is synthetic, it is still only a singular proposition. **For insofar as we here take account merely of the synthesis of the homogeneous (i.e., the units), the synthesis can here occur in only a single way**, although the *use* made of these numbers afterwards is universal. [Geometry is different in this respect.] If I say that by means of three lines, two of which taken together are greater than the third, a triangle can be drawn, then I have here the mere function of the productive imagination, which can make the lines be drawn greater or smaller, and can similarly make them meet at all kinds of angles chosen at will. By contrast, the number 7 is possible only in a single way, and so is the number 12, which is produced through the synthesis of 7 with 5. Such propositions, must be called not axioms (for otherwise there would be infinitely many axioms), but numerical formulas."

The first reason Kant gives for there being no axioms of arithmetic is contained in the underlined passages. He first claims that some numerical relations have one desired property[8] of axioms, that they are evident. But because they are not universal, they cannot be axioms. It is somewhat unclear what Kant exactly means by universal here; I take it as meaning that particular propositions of arithmetic are insufficiently general to count as axioms. The bold passage, concerning how synthesis only occurs in one way in arithmetic rather than in many ways as in geometry seems to support this hypothesis. The second reason for the absence of arithmetical axioms is found right at the end of the quoted paragraph in the boxed passage. Kant writes that if any of the individual arithmetical propositions were to be called an axioms, there would be hence an infinite number of them, which he regards as unsatisfactory[9].

[8] More properly - Kant thinks that axioms must be self evident. I do not, especially in pure mathematics.

[9] It is not at all clear to me why Kant thought having an infinity of axioms was so bothersome. Perhaps he felt that all of the facts of arithmetic would thereby become axioms and hence there would be no (interesting) theorems left to produce from them. Another factor possibly at work would be some sort of psychological claim about us being unable to deal with actual infinities in their totality, as we find Kant worrying about in the antimonies of pure reason. I note in passing that modern mathematical systems (for instance, the propositional calculus as presented in Machover 1996) are often said to have an infinite number of axioms, usually specified by one or more axiom schemata.

Let us thus finish the section on Kant's account of mathematics by examining the puzzling line about arithmetic and space. It seems to be related to the principle of the axioms of intuition (B 202), namely that "all intuitions are extensive magnitudes[10]". I take it that because Kant thinks that arithmetic deals with magnitudes in intuition, it thus must deal with extensive magnitudes. He explains at A 163 / B 203 why extensiveness involves space (italics in original):

> "Extensive is what I call a magnitude wherein the presentation of the parts possible (and hence necessarily proceeds) the presentation of the whole. I can present no line, no matter how small, without drawing it in thought, i.e. without producing from one point onward all the parts little by little and thereby tracing this intuition in the first place. And the situation is the same with every time, even the smallest. In any such time I only think only the successive progression from one instance to the next, where through all the parts of time and their [addition] a determinate time magnitude is finally produced. Since what is mere intuition in all appearances is either space or time, every appearance is - as intuition - an extensive magnitude, inasmuch as it can be cognized only through successive synthesis (of part to part) in apprehension. Accordingly, all appearances are intuited as aggregates (i.e. multitudes of previously given parts); precisely this is not the case with every kind of magnitudes, but is the case only with those that are presented and apprehended by us as magnitudes *extensively*."

In other words, because the process of generating a magnitude in intuition requires a <u>succession</u>, it necessarily involves space. The succession is what allows us to build up a presentation of a whole (even an arithmetical one) by considering its parts in turn.

We have now seen how Kant conceives of mathematics by examining each of the branches of mathematics he discusses and how they relate to one another and to other general concepts, such as that of space.

<u>Section 3 - Kantian Similarities and differences</u>

We have seen some hints towards how Kant thinks of the difference between metaphysics and mathematics and also something of his conception of the similarity. In this section we shall explore this, focusing primarily on the differences.

At A 718 / B 746 of the *Critique* Kant gives one explanation of why he thinks mathematics and metaphysics are alike. Both are to involve synthetic, a priori propositions:

> "At issue here are not analytic propositions (in this the philosopher would doubtless have the advantage over his rival); rather at issue are synthetic propositions - and such, moreover, as to be cognized a priori."

There is also another way in which metaphysics and mathematics interact; this concerns the notion of space. Kant explains that the general notion of

[10] The German here is "Größen". I flag this as this seems to be one possible place where the translation I am using is a bit funny.

space is to be elucidated by the philosopher in the field of metaphysics. This general notion is then presupposed by the geometer. In this sense, there are metaphysical presuppositions of mathematics for Kant.

But Kant was also concerned with how mathematics and metaphysics ought to be unalike. There are several ways in which Kant thinks mathematics is unlike metaphysics. An interesting one is found at A 424-425 / B 452-453, where he explains that no false assertions can remain hidden in mathematics:

> “Only to transcendental philosophy, however, does this skeptical method belong essentially. In any other field of inquiry it may perhaps be dispensable, but not in this one. Using this method in mathematics would be absurd; for there are not false assertions can hide and make themselves invisible, inasmuch as the proofs must always proceed along the course of pure intuition and, moreover, by a synthesis that is always evident.”

One need not be skeptical in mathematics, as mathematics deals with what is evident and intuitive. But intuition does not play a role in metaphysics. Kant explains at A 711 / B 739 that the reason for the *Critique* is precisely this.

> “But where neither empirical nor pure intuition keeps reason on a visible track - viz., in its transcendental use according to mere concepts - there reason needs a discipline that will subdue its propensity towards expansion beyond the narrow bounds of possible experience, and that will keep it away from extravagance and error.”

This passage also gives the main difference between mathematics and metaphysics for Kant. Metaphysics is transcendental and uses mere concepts. To finish the exploration of this Kantian distinction, let us now look at what Kant means by this phrase.

Transcendental is explained at A 56-57 / B 81 as the adjective one uses to mean to do with “objects in general” and at A 295-296 / B 352-353 to mean going beyond the boundary of all possible experience. “Mere concepts” in turn refers to the lack of the influence of intuition in the area of metaphysics. At A 68 / B 93 he explains that conceptual thinking and intuition are actually diametrically opposed, and further, exhaust all human cognition:

> “Apart from intuition, however, there is only one way of cognizing, viz., through concepts. Hence the cognition of any understanding, or at least of the human understanding, is a cognition through concepts; it is not intuitive but discursive.”

This passage tells us the final fact of importance that distinguishes metaphysics and mathematics for Kant. Mathematics is not discursive and metaphysics always so. Philosophical “demonstrations”, such as they are, thus acquire a different name to emphasize the fundamental difference between them and those of mathematics. Kant names them ‘acroamatic proofs’ at A 735 / B 763.

We have thus seen the some of the differences and similarities between mathematics and metaphysics for Kant. In the next section, I shall present some possible criticism of Kant’s general project along these lines in a Leibnizian vein.

<u>Section 4 - Pseudoleibnizian Criticism</u>

This section will briefly discuss how a Leibnizian might have responded to Kant[11]. Six main points can be made, three metaphysical and three concerning mathematics.

Let us look first at the metaphysical points, then move onto the mathematical ones.

First, a Leibnizian is going to deny that humans always engage in metaphysical speculation. This entails in turn that a radically different conception of metaphysics is possible to a Leibnizian. It certainly would require a different sort of restriction if only certain people were to be metaphysicians. They will likely remind Kant that many people do not “think of thinking.”

The second concerns the application of metaphysics to understanding the phenomenal world. While Kant argues that such an application is impossible, a Leibnizian can simply play the “god card” that Kant wants to avoid. Kant is <u>right</u> to say that saying that “we can make this application because of the preestablished harmony” is not saying anything of any consequence, but he is simply begging the question against the Leibnizian. Kant has to <u>show</u> why the preestablished harmony (or other similar conceptions) are unsatisfactory[12].

The final metaphysical objection is the collapse of the distinction between metaphysics and epistemology. A Leibnizian is going to object to the claims of Kant about our cognitive faculties matching experience and concluding that the world in itself is unknowable. She will object, telling us that there are things about the world that are known, but not by experiencing them, for example about momentum conservation. (Remarkably, so will a Newtonian.)

Let us move on to the mathematical issues, then. First, the issue concerning how no false propositions can hide in mathematics. This thesis would be somewhat difficult for a Leibnizian agree with, as Leibniz seems to have recognized that when one proceeds by *reductio* one can choose the premise to reject.

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[11] There is one whole area of issues here that I am deliberately ignoring as it is too complicated to deal with here. This is the issue of the indivisibility of monads.

[12] As far as I am concerned, a suitable naturalistic answer to this sort of issue (like the one Kant seems to want) was not available until Darwin. Again, however, this is another story for another time.

Second, a Leibnizian is simply going to deny Kant's thesis that there are no axioms in arithmetic. Leibniz (1990 [1765]) repeatedly states throughout that arithmetic is no different in this respect from geometry. He even attempts to give an axiomatic derivation of '2 + 2 = 4' (which almost works, too). Hence any of Kant's comparisons of the principles of arithmetic and those in metaphysics will fail. For instance, when he explains that metaphysics too cannot have axioms, because it is not universal in the right respect, he is drawing on a principle he is also using in the account of arithmetic. This concern thus affects both halves of Kant's project.

Third, (and I understand this was actually raised by some Leibnizians) concerns the issue of "intuition" and similar concerns. Kant says, as we saw, that to deal with a line we have to draw it in thought. Does that mean picturing it one's mind's eye? If that is the case, and it certainly seems that way based on the use of the verb "draw", how does this imagistic way of doing things work? We do not have the ability to visualize certain kinds of things, and further, one can do the geometry without the drawing (mental or otherwise). This is exactly a point that Leibniz brought up against Locke.

Now that we have seen a sketch of Kantian metaphysics, a fragment of Kant's philosophy of mathematics, something of their interrelation, and finally some objections to these accounts, we can now understand the subtitle for the paper. Kant's distinction is well grounded within his conception of two disciplines. However, as the Leibnizian remarks point out, Kant is missing a few fundamental pieces in his understanding of mathematics and metaphysics, and by extension the notions that tie them together are dubious as well.

References

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[13] Aside: If the reader of this paper has not seen this book, I recommend that she read it. It is a delightful little paperback history of mathematics.