UBC Philosophy GSC — Keith Douglas

<u>Keith's current research interests, etc.:</u>
My main research area is philosophy of science and technology. Current projects include ontology of computing, law statements vs. methodological principles in chemistry, the relationships between psychology, cognitive neuroscience and epistemology, the 10 or so "boundary problems" and, finally, the public perception of science, especially including the so called "science wars."

<u>Abstract:</u>
This presentation concerns a way of doing all branches of philosophy called "exact philosophy". Its importance both for philosophy itself and other fields will be sketched and its potential hazards will be discussed.

<u>Exact Philosophy - what it is, why it matters</u>

This presentation is going to be a bit unusual in three respects. One is that it is a piece of metaphilosophy, which is somewhat strange. Second, I have prepared it as something of a lecture/discussion provoker rather than a paper *per se*. Third, it was originally also supposed to be an introduction to my interests in philosophy. I do not think I have met this latter goal at all. However, I have instead tried to explain a metaphilosophical goal of mine that if we can agree to something of its goal we can build a foundation for future philosophical interaction. Hence perhaps with this bedrock, I will be better able to present my views on various other issues at a later time.

I thus will divide this presentation into basically four parts. First, I will provide a programmatic definition of exact philosophy, and explain the origin of the term as I know it. Second, I will explain what exactly is and is not involved. Third, I will briefly talk about several examples. Fourthly, I will discuss possible criticism, both justified and not of the project I have sketched.

A warning, before I begin, however. Part of this presentation is necessarily mathematical. This hopefully should not put anyone off; in fact, part of the goal of exact philosophy is precisely to explain why math matters. If the <u>mathematics</u> of what I am talking about is too complex, let me know - it shouldn't initially be a problem, as we shall see. I will not be teaching any of the math except very indirectly; the presentation is, after all, about how to <u>use</u> math.

Alright. First things first, a definition. I define exact philosophy in two ways, depending on whether one is talking about a goal or a method. The goal of exact philosophy is to render in as precise a way as possible our hypotheses, definitions, and so on. An exact philosophy would make use of mathematics to this end. We shall see how this to work in due course. The <u>method</u> of exact philosophy involves concept honing, and in particular, developing systems of honed concepts. What does it mean to hone a concept? Well, many of you no doubt remember those “baby logic” classes that we probably all took at some point. Do you remember wondering about those unfortunate predicates like “is hairy”?

Well, the solution to that is going to be explored in this presentation. Suffice is to say that “is hairy” must be refined and processed to make it more usable.

The term “exact philosophy”, for those who care about such things, seems to have been first used extensively by Mario Bunge, one of my undergraduate philosophy professors. One can find a discussion of this in the introduction to the collection of papers at the first exact philosophy conference, which as it turns out was several years before I was born! So to me, I am not talking about a very new subject.

So, why do we care about such things? How do we do it? Let’s start with a simple example of cleaning up a sentence. Later we shall see how to clean up whole collections of them.

“Bertrand is my young, fat, stupid, uncle.”

The previous sentence deliberately confuses at least two meanings of the word “is”, for one thing. Admittedly, Bill Clinton didn’t bring it up at the right time, but “is” does have several meanings, as I am sure you are aware. Let’s briefly review these to show the problem with the above sentence.

1. identity. In the above sentence, “Bertrand” and “my young, fat, stupid, uncle” pick out the same object, so the “is” here likely represents that of identity. We normally write this Bertrand = my young fat stupid uncle, or sometimes Bertrand ≡ my young fat stupid uncle. There are differences in meaning here between the two symbols; however, the literature does confuse them. Some authors use one for equality and one for identity, and so on. This does not matter much, so long as they are spelled out in advance or follow one of the established traditions explicitly. This is perhaps one of the weakness of the exact approach, that symbolism does vary a lot. I note however that this infects meanings of words in ordinary language as well, and further that exact does not necessarily mean symbolic.

2. Set membership. The sentence suggests several memberships. Nominalists should hold off their worries until later. “Bertrand” is in the set of young things, the set of stupid things, and the set of uncles-of-me. Presumably he is also in

the set of uncles. We normally write "Bertrand $\in$ uncles", and similarly for the others. I will explain more about matters of set theory a bit later.

3. Then we come to the is of predication. We predicate fatness of Uncle Bertrand. We could presumably also predicate uncleness of Bertrand, which allows me to make another point of importance to exact philosophy. In order to keep one's levels of reality straight, it is important to distinguish between properties and predicates. I remember reading some of the philosophy of perception literature and noting this confusion. This is important, because one of the quibbles people worry over is whether there are "negative facts", or the like. This worry dissolves on remembering that predicates represent properties, in all fields except mathematics, which might possibly explain why some logicians have confused the two. Let's look at a mathematical analysis of predicates to sort out this confusion a bit better.

Predicates can be looked at mathematically in several ways; one way is to look at them in terms of a function of the cartesian product of several classes of things.

Suppose we want to analyze "is green". A first analysis based on my knowledge of perception is as follows:

$$G: B \times S \times I \times W \rightarrow B$$

is green, therefore, is a function from bodies, subjects, illumination levels, and something to do wavelengths of ambient light to the class of bodies. The fourth argument of the function is probably itself a function of all the different wavelengths in the room; further investigation would show this.

This way of looking at predicates allows us to draw several interesting conclusions, as well. One is that Kuhn's (1962) thesis, ever so popular these days, on the radical incommeasurability of theories, is false. I'll make a brief digression as to why to show the power of going at this argument armed with more than ordinary language.

Kuhn might have claimed that there is a radical break between, say, special relativity and Newtonian mechanics, and that hence "adopting" one theory rather than the other is something like a conversion experience. But there are exact methods for determining what something refers to. Many theories of reference exist; here's a use of a simple one to show the radical version of Kuhn's thesis is false.

Predicates can be represented as a function from a cartesian product of sets to another set, as we have seen. In particular, the mass predicate in newtonian mechanics is a function from the cartesian product of bodies with that of mass units to nonnegative integers. In short:

$$M_n : B \times U_M \rightarrow \Re^+$$

The mass predicate in special relativity also involves the reference frame.

$$M_r : B \times F \times U_M \rightarrow \Re^+$$

Of course, the units aren't part of what the predicate refers to; so we can build the reference classes of each predicate.

$$R(M_n) = B;$$
$$R(M_r) = B \cup F$$

So Kuhn is wrong; they are comparable. In particular:

$$R(M_n) \subset R(M_r)$$

Only by examining what a theory refers to can we determine whether theories refer to the same things or not. Ordinary language analysis of the same problem is difficult, to say the least. I do not know of any other way to specify more clearly what one means in this sort of context, so we have gained a positive result here.

4. Finally, the "is" of set inclusion. This is not illustrated at all by my example sentence, so I will give another easily understood example. Again, nominalists should hold their worries until later. "Mammals are animals." This says that the class (set) of mammals is a set included in the set of animals. This is normally written "mammals ß animals".

I've shown some simple cases where some exact tools have clarified meaning. Let's look at another reason to prefer exact statements to fuzzy ones, testability. Suppose you're a philosopher interested in the social mechanisms of rumour diffusion. How do you state your hypothesis of, say, rates of diffusion in order to maximize testability? Mathematics provides another tool, the differential equation. I borrow this example from Stewart 1991.

Suppose you think that the rate of spread of a rumour is proportional to the product of the fraction of the population that has heard the rumour and the fraction who have not. This is an exact hypothesis, and indeed, a quantified one. Stating it this way does allow us to better test the hypothesis. We write:

$$\frac{dy}{dt} = ky(1-y)$$

Here y is the fraction of the population that has heard the rumour; dy/dt is hence its rate of change with time; k is a constant. Don't forget that latter bit!

You can solve that equation, and use it to more easily produce predictions of your model. Solving the equation (I won't go through the details; I am not giving a course in differential equations, after all) one obtains, choosing y nought as the constant of integration:

$$y = \frac{y_0}{[y_0 + (1 - y_0)e^{-kt}]}$$

Right away we can take the limit as t = infinity and see that the fraction of the population that has heard the rumour goes to y nought over y nought, or 1.

Notice also that this equation could conceivably come up in another field; in fact, the idea of a philosopher interested in rumours is sort of odd. But if one did find something that we could model in the same from as the differential equation above, one would have already saved oneself a bit of work, as the equation is already solved. This is related to the issue of correspondence rules, which I will talk about later. Note we have to measure or find from elsewhere several values in order to use this model for predictions; this is the case with any of them we choose to build. A whole other paper could be written on measurements in philosophy, but that's for another time. Note that what we have done precedes that - can't measure before knowing what to measure!

Here we have greatly enhanced our power of testability by using something relatively modest mathematically. If you are asking what is the point of testability in philosophy, hold that thought. I will return to it when I deal with objections in the final section of this discussion.

Here's another example: a very tiny piece of exact ethics, based on Bunge's (1996) work. One objection to exact philosophy often is something along the lines of "well, <u>my</u> subject can't be quantified!" Two things can be said to that. One is that there are non quantitative branches of mathematics, which I'll be using in just a moment, and two: how do you know? Whatever you are dealing with comes in degrees, no? So there's a way in principle to quantitate it. If you can't figure out one that can be tested, though, don't bother. But more on this later. I also note that this is not a theory per se On to an objective, exact, though somewhat preliminary sketch of utility.

The intuitive idea behind this is that something is useful or beneficial if and only if it meets some need or desire of someone. I will take the notions of need and desire for granted;

they are undefined by the proceeding; one might have an account (or even a theory!) of them elsewhere - or not. So we say that the utility U of object x for animal/social group y is the collection of needs (N) or wants (W) of y that x meets or satisfies. This can be written:

$$U(x,y) = \{z \in N \cup W \mid Mxzy\}$$

where Mxzy is short for “x satisfies need or want z of y”.

This allows us to set up an exact preference relation for a given subject y.

$$a \geq_y b =_{df} U(b,y) \subseteq U(a,y)$$

This says “a is preferable to b if and only if the utility of b for y is included in that of a.”

This has the advantage that the preference relation inherits the properties of the set inclusion relation, so we don’t have to go out and “make another relation” ourselves.

From here many refinements suggest themselves. One can revise the above notion relatively easily to elucidate action preferences.

This would proceed as follows. Call the utility of the consequences of an action *a* as u1 ... uk; call the disutility of the consequences of an action *a* as d1 ... dl. Form two sets big U and big D of all the little Us and little Ds. We can do the same for the action b. We now have constructed four sets U(a), U(b), D(a) and D(b). Put:

$$a \geq b \text{ iff } [U(b) \subseteq U(a)] \wedge [D(a) \subseteq D(b)]$$

Action a is preferable to b if and only if the utility of b is included in the utility of b and the disutility of a is included in the disutility of b. Note that there are no numbers involved. Mathematics need not be quantitative. More on this later.

Now, since this is an isolated statement, not much of anything comes out of it; the real strength of exactness does in fact come from systems of statements, theories, which is what I turn to now.

<u>Building systems of ideas</u>
This section is the most difficult to explain, and the most rewarding when done well. It is also the part I have done least myself, but I see its value and want to communicate that viewpoint to you as part of this presentation.

First, I must confess to a little gripe I have with some terminology. I talked a bit about <u>theories</u> before without explaining what I meant. Given the subject of the previous part of the presentation, this might have seemed a bit odd. But there was a reason for it. This has to do with my gripe. I am not fond of the current sloppiness attached to this word. With so many weird uses of the word “theory”: psychoanalytic “theory”, critical “theory”, feminist “theory”, relativity theory, group theory, and so on, one can begin to wonder what this word means.

I am going to propose that we philosophers use the term as used in mathematics and physics, and to a lesser extent in the other natural sciences. It seems that the social and mixed sciences (for instance, psychology) have been partially guilty of the sloppy use as much as those in the humanities.

A theory is a hypothetico-deductive system. Axioms, definitions, and their logical consequences. (Logic? Are there any who are up turning their noses at this idea?). The reason I use this exact definition is that it makes “theory” have a precise meaning. Now, of course, anyone is welcome to use words in new and different ways, but they should attempt to make them clear. As far as I can tell, the use of “theory” now basically means “any account whatever”, which is so broad as to be useless. I’ve even seen single untestable statements called theories. Now if that isn’t playing Humpty Dumpty, I don’t know what is[1].

The use of this word and the discussion of theories in the above (as far as I am concerned correct) sense allows us to look at another strength of exact philosophy. As some of you may be aware, one of my current projects is “boundary areas”. Now, I actually think there is no philosophy/science “Berlin Wall”, and that’s in part because one can build “useful” theories which are both, such as (perhaps) a general account of spacetime, or a descriptive epistemology based on neuropsychology. Both of these are being done, and at least the latter has been done in the form of a hypothetico-deductive system on at least two occasions. In other words, exact philosophy allows us to productively do that fashionable activity, interdisciplinary work, which of course is something that we philosophers tend to do in spades these days.

Now, there are two kinds of theories. Logico-mathematical, or formal, and factual. If you are interested in logic as one of our interests within philosophy, you’ve no doubt met this kind of theory before. Here the axioms and definitions, and hence the theorems (their logical consequences) don’t refer to anything in the world. I am of course taking a non-platonistic stance. If you’re a platonist, you think that logic and math are discovered, which for the present discussion is fine; we can

[1] For those of you who do not remember Lewis Carroll’s Humpty Dumpty, he basically said that he uses words mean exactly what <u>he</u> wants them to mean, no more and no less. Plato pointed out that this doesn’t work quite some time ago, which was Carroll’s point.

talk about ontology of math later. The point one has to grasp here is that formal theories don't refer they don't refer to objects like photons, rational agents, or societies.

These are referred to by factual theories. Hence if you are wanting to refer to the factual world with your theories, as all exact philosophy would save in logic itself, you must set up *correspondence rules* between your axioms and the real world. This can be done by setting up a little list of primitives of the theory.

I remark that one need not explicitly write down:

PRIMITIVES
...
AXIOMS
...
DEFINITIONS
...
THEOREMS
...

The degree to which one structures in <u>that</u> way is subject to situation and taste. I for one like a more flowing approach, but of course still one in which all of the above are reasonably clear.

Before I give an example of a theory-fragment, I would like to point out also that one need not define or refer every concept in the theory explicitly. In Newtonian mechanics, one *implicitly* defines mass by the theory. This is another value of theory building; one can avoid the Aristotelian trap of trying to define everything without leaving some key concepts vaguely stated. This is routinely done in mathematics, especially, where an object is given several properties. "Let there be an ordered pair thus and so. We call this a group." And so on. How exactly one wants to balance primitive and defined notions is subject somewhat to taste and also of course depends on what the presuppositions of the theory are. If you are working in, say, political philosophy, you may refer to previous works in, say, ethics and philosophy of law, and take those as given - even if to show that they have undesired consequences.

So here, I will give elementary probability theory, and give it a very brief (but exact) ontological interpretation.

Primitives: U, a non empty set, a function P defined on S(U), the set of all non-empty subsets of U and taking real values. Presuppose ordinary logic, set theory and arithmetic.

Axioms:

P1: $\forall A_{S(U)}[P(A) \geq 0]$

P2: $P(U) = 1$

P3: $\forall A_{S(U)} \forall B_{S(U)} [A \cap B = \varnothing \rightarrow P(A \cup B) = P(A) + P(B)]$

One can read the last axiom loosely as "the probability of disjoint 'items' is the sum of the individual probabilities."

So far I have not <u>interpreted</u> the theory for you; to show briefly that it has a possible ontological interpretation, one needs only two correspondence rules. These are:

Let U be a set of possibilities (i.e., possible facts) of a kind; let P(A) be the weight of the possibility of A. This ontological interpretation can be viewed as a (more useful) "big sister" to the qualitative theories of possibility, the various modal logics.

Before proceeding to the limitations and pitfalls of the exact method, I should like to point out there are numerous other benefits that I have not had time to go into; I'd be happy to discuss these later, but in the interests of making this presentation more manageable, I've left them out.

<u>Pitfalls & Limitations of the Exact Method, Perceived & Correct</u>

There are several limitations and dangers of exact philosophy, which I will address in due course, but there are several objections to it that I find spurious. I would like to deal with these first, then I will examine what I take to be the legitimate criticisms and shortcomings.

The first comes from the historians of philosophy, of which there seem to be an awfully large number these days. The objection goes something like this: You've talked endlessly about sharpening our concepts and creating new ideas in exact form, and all of that. That's fine, so the objection goes, but I am interested in recreating the argument of X against Y's account of Z, and so on, and X was notoriously obscure, or something along those lines. How does exact philosophy allow me to do research in the history of the discipline?

I have two answers to this, one slightly facetious, one serious. The silly one first. Well, at the very least you are demonstrating one ways in which exact philosophy is valuable! By adopting it as part of one's metaphilosophy one can reduce squabbles over what one said when future historians of philosophy are intellectually exhuming your work. The serious answer is that the same tools can be used to look at the great (or not so great, as you may find out!) philosophers of the past. There's a danger here, of course, which I'm sure you will all see, but the power is still welcome. For instance, in the collection of papers from the proceedings of the first exact philosophy conference, there is a discussion of Plato's account of forms as discussed in the Phaedo. (Castañeda 1973) An mathematical analysis is made (remember that mathematical does not mean quantitative!) and a defence of Plato against some critics is produced thereby. I will not go into the details, but the very fact that it has been done shows that the general thesis is false.

The second objection comes from the current cluster of people who are vehemently opposed to mathematics and logic, or regard it as “phallocentric”, “bourgeois”, or any number of other things. I will not argue against these views explicitly now, but all I can ask for is evidence that their thesis is correct. As far as I can tell, there is none whatever. I have provided references to this effect in the bibliography. [Bibliography note: See, e.g.: Patai and Kortege 1994; Gross, Levitt, and Lewis 1996; Sokal and Bricmont 1999 and others] In fact, exact tools can be used to elucidate the reference classes of concepts, as I discussed briefly earlier. These can show that, say, that there is nothing phallocentric about numbers. The suggestion that the theory of reference is biased will be met with a similar request for evidence. [Note in written copy: If asking for evidence is viewed as holding an undesirable or “just another” bias, I have nothing further to say to you. I think it should be obvious why.]

The third objection I shall reject as spurious concerns the worries of those that feel axioms will “entrench” their presuppositions and make change of points of view impossible. On the contrary, being explicit about axioms is a good thing. If we aren’t clear about our starting points, often one can argue back and forth without any real hope of resolution because one’s presuppositions aren’t spelled out. Using axioms and references to presupposed other theories overcomes this limitation.

A fourth and final objection concerns basically the Nietzschian objection to Spinoza[2], or (on a slightly different line) objections the empiricists and Kant raised against the Cartesians and the Leibnizians to “theory” as I have presented. Basically one has to make sure one follows what Bunge called (Bunge 1999) his exactness decalogue, or something similar, which is ten methodological points to make sure one is not “making castles in the air”. I forget who coined that phrase with regards to the excessive rationalists[3], but it is apt. Here they are, in my slight rewording.

Exactness Decalogue

[2] Thanks to Professor Sikka for reminding me about the Nietzschian objection, and Professor Carson at McGill for the bit about “castles in the air”.

[3] It might be inferred from my previous remarks that I am not an empiricist either, which might suggest to some that I am some what epistemologically unusual. Well, yes. But that’s off topic.

1. Any inexact and reasonably intelligble idea can be exactified.
2. Given any exact idea, it is possible to construct an even more exact and powerful one.
3. Always prefer the more exact of two roughly otherwise-equivalent ideas.
4. The best conceptual analysis is synthesis (i.e. theory building).
5. The importance of an idea is (directly) proportional to the number of ideas it can be related to in an exact manner.
6. Do not expect a single exact concept, proposition or theory to solve all your problems.
[This one should be the one that Spinoza should have paid a bit more attention to, if I may speak anachronistically]
7. Do not use exactness to bully or to fool.
8. A good idea, even if somewhat fuzzy, is to be preferred over an exact but pointless or false one.
9. Do not pursue exactness at the expense of substance.
[Some modern logicians seem to have fallen afoul of this one]
10. Do not crow over any exactification, for eventually it may be shown to fall short of even higher exactness standards.

With these in mind, I think one can put many of the worries to rest. I would like to point out another few of the shortcomings myself, then I will turn further possible objections to the floor.

First is the quantitative issue. I've talked about set theory as being nonquantitative; there are in fact lots of nonquantitative branches of mathematics. Even Euclidean geometry basically is; the concepts of number it uses are "naive" - i.e. they are sort of every day notions, and for that branch of mathematics, that's fine. Topology is another qualitative branch of mathematics. Unfortunately, and this brings me to my next point, is that one can abuse mathematics and mathematical terminology to give discourse a "high falutin'" look when it is in fact largely devoid of content. I mention topology in this context, as the psychoanalyst Lacan was notoriously abusive of topological terminology. Let me stress again - if you want to use mathematical tools and concepts, one has to show their correspondence to the objects of study. Perhaps topology does have something to do with neurosis[4], but I doubt it - at least pending evidence to that effect. I tried to do this in my various examples, particularly in the example of part of a theory. I am basically stressing the importance of numbers 7 and 8 on the decalogue, as these are the most difficult ones when dealing with the work of others.

[4] If the reference here is obscure, it doesn't matter terribly. Suffice is to say that Lacan thought that there was some connection between a torus (roughly, a donut shaped object) and the "structure of the neurotic". See Sokal and Bricmont 1999 for criticism.

Also, it should be noted that the branches of mathematics used in any particular case is up to you, the creator of the system of ideas. It is of course true that certain kinds of math are more suited to certain kinds of things, but hey, if you feel really ambitious you can even invent a new branch of math to solve your problems! Newton did this. (This is something of a joke, as there are so many branches of mathematics that have been created anyway - chances are you could learn one that will meet your needs.)

We've now seen a few remarks on the problems I perceive with this enterprise; I'd now be happy to discuss. Thank you.

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