Armstrong’s Metaphysics and Philosophy of Mathematics

Introduction

In this paper I will do four things. First, I shall briefly discuss Armstrong’s (1997) metaphysics with special attention to his mereology. This shall lead into a discussion of his philosophy of mathematics, which is strongly associated by means of an Aristotelian realism to this metaphysics. Throughout, I shall attempt to raise difficulties with metaphysics and philosophy of mathematics in the light of what might be called Leibnizian moves (and one comment based on apparently more sophisticated chemical understanding). I shall briefly compare his position to Maddy’s 1990 defense of what she calls physicalistic platonism and attempt to use her set theory to solve some of the worries I had with Armstrong’s accounts. Finally, I shall conclude with brief remarks on what I take the relationship between metaphysics and philosophy of mathematics to be. (Warning: The present paper can thus be read as intertwining two braids, one metaphysical and one mathematical, and sometimes the braids are thus very closely tied together. Also note: many of the remarks here are programmatical, and suggest further investigation.)

Section 1: Armstrong’s Metaphysics

In this section, I shall briefly sketch out what I take to be some of the essential features of Armstrong’s (1997) account of general metaphysics. I shall place his discussion of mereology and its relation to classes in this context.

The central concept of this work of Armstrong’s is that of the state of affairs. A state of affairs (pg. 19-20) is ‘what is the case’, or (roughly) ‘fact’. There are three kinds of states of affairs: strictly atomic, loosely atomic, and molecular. Strictly atomic states of affairs are at the ‘lowest level of being’ and cannot be further analyzed. Armstrong recognizes that there may not be such things at all. Loosely atomic states of affairs are those relative to a given context. Molecular states of affairs are thus made up of atomic states of affairs in this loose sense. These are conjoined by a special metaphysical relation of conjointness. More on this later. These three concepts are important and will play some role later.

Next we must examine Armstrong's account of properties. This is important for the philosophy of mathematics discussion in the sequel, so it bears some attention. Armstrong first rejects (pg. 20-21) two forms of nominalism. These both involve denying the existence of universals, and in the strong form, the existence of properties and relations. He refers the reader back to his earlier work on this subject (Armstrong 1978) and summarizes the findings from this as follows. The moderate form of nominalism denies the existence of universals, but admits properties and relations. Strong nominalism admits none of these. Armstrong also (pg. 21-22) rejects an extreme form (Platonism) of realism with regards to properties, claiming the name Aristotelian realism for his viewpoint instead. This position holds that universals, properties, etc. are real features of the world of becoming and rejects Plato's realm of forms. Finally, Armstrong also (pg. 27) tentatively rejects disjunctive and negative properties but leaves them somewhat open in the light of scientific findings like the Pauli exclusion principle (pg. 28). Most important for our purposes, however, is his acceptance that there can be conjunctive universals. If a particular instantiates both the universal F and the universal G, Armstrong sees no reason not to say it instantiates the universal F & G. The acceptability of this to me turns critically on how we are to understand "&" in this context, which brings us to Armstrong's first discussion of wholes and parts. The logical conjunction operator ("&") is related to his "+" operator of mereological composition, of which we shall see more about shortly. He says there (and repeats elsewhere) that he views mereology as an extension of the logic of identity.

> "One is whole/part, the other is overlap. Mereology, which deals with these notions, may be thought of as an extended logic of identity, extended to deal with such cases of partial identity."

This is our first point of contact between philosophy of mathematics and metaphysics, and it is a mysterious one. It is not clear how the above remark is to be taken. Armstrong gives an example of a rowhouse, where those in the centre of the row share two walls with their neighbours on either side. He says that this sort of relation is elucidated by mereology. In order to investigate this claim, let us examine what mereology is.

Two kinds of mereology (with a possible third kind ‘half way’ between the two) must be distinguished. I shall digress briefly to explain this distinction, and then we shall return to examine Armstrong’s remarks in that light. The primary kinds of mereology are mathematical and metaphysical, with perhaps a ‘formal’ mereology half way between the two. (This is important, because Armstrong’s account may be yet another kind, or a mixture of several of these.)

Mathematical mereology is a branch of pure mathematics. It studies a particular kind of ordering relation on arbitrary structures. This kind is exemplified by many of the systems explored in Simons’ (1987) *Parts: A Study in Ontology*. It is, however, sometimes difficult to tell which systems in this work are intended to be which kind of mereology. Uncharitably, virtually all of them are, as they make no <u>explicit</u> reference to nonformal things. (This is very often a mistake of presentations of exact theories in the sciences - the correspondence rules are not made explicit.) I am sure, however, that many of the authors of these systems thought they were describing the world, to which we now turn.

Metaphysical mereology is a different enterprise. These theories attempt to elucidate the actual part whole relations <u>in the world</u>. Moreover, they are often part of a more general ontology. We shall see this in a moment, but first a slight digression to point out that there may be use for at least a third kind of mereology.

Simons’ 1987 briefly mentions Rescher’s discussion of the mereology of sentences. I have not read Rescher’s work on the subject, but it strikes me as at least *prima facie* plausible that a different kind of part-whole relation might work with certain kinds of conceptual objects (e.g. sentences, languages, computer programs in themselves, etc.) This would stand to reason, as the parts of a sentence (for instance) are conceptual, yet the parts and wholes are in some sense <u>of</u> something; a purely mathematical mereology studies the relation in itself and is not concerned with any relata at all.

Metaphysical mereology is taking the above one concretizing step further. It can be viewed as an attempt to model actual part-whole relations in the world[1] (i.e. involving <u>actual</u> relata). As such, it can be embedded in a general ontology.

The goal of the general ontology in question and in particular what theorems one wants to prove, affects its structure. If, for instance, one wants to prove some general conservation laws, one possibly needs a null individual in one's mereology in order to so. Other ontologies without this goal do not need this fiction added to them. (I will defend the worries over realism that this raises in due course when I discuss the role of fictions in factual theories a bit later.)

We must then ask which kind of mereology Armstrong is doing, if any of the above. A possible clue comes in the discussion of classes, chapter 12 of his account. This section suggests that we should study the features of relations between classes he discusses and their metaphysical import. Let us focus on a discussion of the conjunction operation and four metaphysically worrisome features of it.

As noted above, the conjunction operation is an operation that takes place between states of affairs and also one that takes place between universals, and 'bundles' the states of affairs into a higher order one. It can also bundle universals into another universal. 'Conjunction' thus is a mereological operation for Armstrong, <u>and</u> in some sense, a class-theoretic one, similar to the usual class-theoretic 'union' operation. Let us look at other class theoretic operations.

Mereological intersection is not too difficult to understand on this scheme, though he does not explicitly discuss this anywhere.

[1] A good parallel is the two kinds of geometry in Bunge 1999. Conceptual geometry (e.g. Euclid's) plus suitable interpretation (e.g. line refers to light ray) refers to real things and thus 'becomes' a physical geometry.

Representing it, however, has a danger of circularity. In other metaphysical mereologies, the formal system is ontologically distinct from the system(s) it (with suitable correspondence rules) purports to describe. On Armstrong's scheme, classes and wholes are interchangeable (because each class has a corresponding aggregate and since classes just are aggregates ontologically according to Armstrong.), so it may be difficult at some point to tell which aspect (formal/factual) is being emphasized.

The second of these concerns his equation of the set membership relation with the proper part relation. Here, it is important to note that proper part is necessary, unless a class theory is wanted where classes are members of themselves. (Under many mereological systems, things are regarded as parts of themselves. In Bunge (1977) this fact is used as part of the implicit definition of the metaphysical part-whole relation.) There seems to be little difference between membership ($\in$) and inclusion ($\subset$) on Armstrong's account. To 'use' an inclusion relation on his account, one has to name a bunch of possible members and assert (or deny) that this new aggregate is part of the whole. This may work metaphysically (it suggests questions of ontological reductionism that I do not wish to address in this paper), but mathematically it is a bid odd. See section 2, below.

A third way in which Armstrong's account is curious concerns his discussion about which classes are aggregates. He tells us (pg. 185) that all classes are aggregates. As we shall see, this leads to strangeness when it comes to classes of mathematical objects; for now let us attend to what the strange features this results in the case of more (conventionally) concrete objects. On page 185 (and elsewhere) he notes that he is willing to allow what he calls unrestricted mereological composition. He's aware (pg. 192) that this can lead to paradox as follows. The worry is due to Gideon Rosen, who (according to Armstrong) suggests that if every object is embedded in its own state of affairs a paradox arises. Each object in a state of affairs is mereologically an atom. So there must be a one-one correspondence between objects and atoms. But given unrestricted mereological composition, there are at least 2^n-1 objects, because for each non-empty class of atoms, there exists the mereological sum of their members. But by Cantor's theorem, $2^n-1 > n$ for all cardinals n greater than 1. Armstrong's proposed solution involves potentiality. I will grant that ontological aggregation can be understood in terms of potentiality, but the <u>mathematical</u> result that results from this is odd. See below in section 2 where this result is discussed.

The fourth worry about the conjunction operation concerns the possible isolation from other parts of Armstrong's metaphysics and the ends of a comprehensive metaphysics taken generally. A good example to explore concerns an earlier discussion (1997, pg. 34 ff) of a methane molecule. Here Armstrong regards a methane molecule as a conjunction of several states of affairs, several atomic and several relational. These are as follows. It has 5 mereological parts: a,b,c,d: the hydrogen atoms in the molecule - thus the states of affairs Ha, Hb, Hc, and Hd. The carbon atom is state of affairs Ce. There are also bonding relations aBe, bBe, cBe and dBe. Thus a+b+c+d+e (where "+" is the conjunction operation mentioned previously) is a methane molecule if and only if these states of affairs occur together. The methane molecule is thus a conjunction of states of affairs.

I object to this account not on the grounds of what has been said, but on the grounds of what has been left out. Chemical examples look deceptively simple to analyze on Armstrong's account, but as we shall see, are really not so easy. A methane molecule has <u>different</u> properties than merely <u>juxtaposed</u>[2] hydrogen atoms with a carbon. It may be rejoined that the bonding relation takes care of that. This is true, some of the emergent (or perhaps, supervenient) properties that methane has are attributable to the bonds. But the bonds <u>themselves</u> have characteristic properties that are at least related to the fact that they are carbon-hydrogen bonds. Not all chemical bonds are the same, even all bonds called covalent, as these are. For instance, typical hydrogen-carbon bonds have a bond energy of 413 kilojoules per mole whereas carbon-carbon bonds have bond energies of 347

[2] It may be suggested that Armstrong's conjunction relation is more than mere juxtaposition. This point is granted for the moment, as the analysis does not rely on it too heavily. It is explicitly examined in due course.

kilojoules per mole (Zumdahl 1993)[3].

We thus must ask if Armstrong's account of relations allow them have properties. Properties of properties are curious metaphysically, and it is not clear if there are nonconceptual ones. (As we will note throughout this paper, Armstrong sort of blurs the distinction between conceptual and actual properties, particularly in mathematical context - of which more below.)

A solution to this is to admit that chemical bonds are <u>things</u>, not properties. After all, they are actually shared electron pairs or something similar. The metaphysical lesson here is perhaps that what appears to be a property or relation may in fact be a thing on further inspection. This is much the dual of the often reported 'caloric' case where something previously thought to be a thing turned out to be a property.

A general root of the previous worries concerns the nature of the conjunction relation, as it is not clear if Armstrong regards it as any more than mereological juxtaposition. The hydrogen atoms in the methane molecule are not merely conjoined to the carbon atom with a relation between them. All the other important features of the molecule are possibly being lost in the "state of affairs" operation. This suggests a further way in which the conjunction operation may be require refinement. Not only is the factual/formal axis problematic (as we have seen above and will

[3] Even more complicated are cases where other atoms in a molecule affect the carbon-hydrogen bonds elsewhere. (For instance, the bonding properties of hydrogen atoms attached to carbons joined to other carbons with double bonds are different than others.) This is important metaphysically, as it makes the analysis into states of affairs quite difficult - where does one draw the line as to what counts as a sum of states of affairs. The character of the bonds in question also depends on whether the compound is in a given solution. Should that count? It becomes clear that any analysis in chemistry of states of affairs is rather involved. Of course, even more complicated things like cells or societies are thus that much more complex to analyze on this account. I suggest that perhaps more clear guidelines as to what is to be regarded as the relative atomic states of affairs to consider in an a given case would be useful. (This is very difficult to pin down, admittedly.)

see more later), the “state of affairs” and conjunction formulation hides the properties that a methane molecule has that are not those of the carbon or the hydrogen atoms. Yes, it is true that (under Armstrong’s scheme) these properties supervene on the juxtaposition. But the issue is that they are properties with both structural features <u>and</u> nonstructural ones and dividing the account up into structure and function (as juxtaposition/state of affairs seems to) misses this. See footnote 2 above for more on this.)

<u>Section 2: Armstrong and Maddy on Mathematics</u>
As we have seen, much (but not all) of Armstrong’s mereology comes out of his discussion of classes. Since it is commonly held that class theory (and in particular, set theory) is an essential part of mathematics, a careful attention to the philosophy of mathematics that can be obtained from his discussions thereof is thereby useful.

We noted above some curious features of Armstrong’s accounts of classes, in particular his reduction of all the class operations to ones of ‘conjunction’ and membership/parthood. Maddy’s account builds in more features of conventional set theory, but runs into a few problems which she attempts to remedy. Maddy starts with concrete collections, saying that we perceive these and from there get the notion of class or set. Armstrong is more atomistic in his approach, saying that classes are pieced together from their members. For both of them, however, classes <u>are</u> concrete.

Classes are thus aggregates of some kind for both Maddy and Armstrong. We must ask then (a) does this allow for an adequate class theory to be developed, and (b) does this meet the needs of other branches of mathematics? Let us deal with these questions in this order.

As we have seen, Armstrong discusses a paradoxical consequence of his views on classes when it comes to Cantor’s theorem, above. As noted, Armstrong tries to escape this problem by using a notion of potentiality. Here the notion of potentiality is curious. Because Cantor’s theorem can <u>potentially</u> (i.e. the world <u>could</u> in some restricted area 'obey' it, but never all at once) be obeyed, does that mean that Cantor's theorem is only true sometimes? This seems to point to a question of how Armstrong's class theory (or what it would be were it to be put in hypothetico-deductive form) is to be understood. Is it meant as an idealized reconstruction of some features of the world (as any scientific theory is), in which case Cantor's theorem is something along the lines of a result in Newtonian mechanics for a perfectly frictionless incline and sphere rolling down it. Or is it meant that a class theory is allowed to be 'perfect' in the sense that mathematical theories can be (up to formal incompleteness, at any rate)? It seems that Armstrong has to make it the former, if classes are to be brought "down to earth" (1997, pg. 185). This of course changes mathematics into an 'empirical' science[4], which would likely horrify many mathematicians. So even concretizing set theory has a problem not merely with mathematical content but even more so with mathematics as practiced. Armstrong can bite the bullet here, but this may turn into a case of a philosopher getting ignored because his prescriptions (no matter how correct and well meaning) are not descriptively adequate - but that is another story for another time.

But what of other mathematical objects? In some views (e.g. Machover 1996; Perlis 1966) set or class theory underlies all of the rest of mathematics. It is sometimes regarded that one can "build up" the rest of mathematics by means of reductive definitions. For instance, $1 =_{df} 0'$, with $0 =_{df} |\{\}|$ and ' the successor operation.. The fact that this leaves open other possible reductive definitions (see Benaceraff 1965) is somewhat immaterial to this viewpoint. One, it is true, cannot say that 0 really is the cardinality of the empty set, or is the empty set, but one can define it that way. (After all, definition is a sign-sign relation, not an ontological identity. See Bunge 1998.)

So classes of classes, for instance, form numbers on this typical view of mathematics. But Armstrong (1997) has a different account of numbers, in particular as universals of a sort, in much the way

[4] Empirical here is put in scare quotes as all what I call factual sciences are both rationalistic and empiricist to some degree.

classes are. “Three” (or grammatically, threeness) is the property in common between ***, +++, and $$$ (as well as of course countless others). There is thus no direct connection for Armstrong between classes and numbers the way there are said to be in most conventional accounts of mathematics. This account of natural numbers seems to work okay, however, it runs into problems[5] when it comes to other kinds of numbers. Let us first look at his account of real numbers. Armstrong claims (pg. 179) that a real number is just a (universal) proportion. π, for instance, is the universal involved in all concrete circles in the relations between their circumference, area and diameter. This appears to be somewhat plausible[6] for π, but what of other real numbers? Compare this with a conventional account of real numbers, for instance in Cameron’s *Sets, Logic and Categories* (1999). This latter account makes use of the well established notion of a Dedekind cut.

At this stage, one may very well wonder if these two (Armstrong’s and the Dedekind cut) notions are capable of being <u>equivalent</u> in any way. First one has to ask whether a Dedekind cut <u>is</u>, or can be “mapped onto” the empirical operation performed. First there is the question of the classes involved. Since (as we have seen) Armstrong has remarked that all classes are at least potentially in the world, what of the classes involved in a Dedekind cut? There are at least two, depending on how reductive one wants to be about them. These two classes are as follows: a partition of the set of rational numbers into two nonempty subsets A and B such that all members of A are less than those of B and such that A has no greatest member. It is not clear how observed ratios play a role in this. Armstrong could say that the Dedekind cut is an idealization, but then would have to answer why the classes involved either don’t refer to things (contradicting his remark about aggregates), or alternatively, why the classes do not appear

[5] I note in passing that this seems to be a very old view Armstrong has resurrected. It seems to be remarkably like Locke’s 1975 (1700) account, which Leibniz (1990 [1765)) criticized.

[6] It isn’t completely, because as Bunge (1998) has noted, <u>no</u> measurement operation will <u>ever</u> give you a transcendental number in question, only (from the perspective of the complete mathematical value) infinitely crude approximations.

to involve anything like his account of numbers. (Real numbers would be radically different creatures than natural numbers.) It also is unclear on this account how two things (recall that a Dedekind cut involves <u>two</u> sets) would resemble one thing.

Second, if the notion that there are classes involved in constructing the reals is rejected, and only accounts of number in the Armstrongian sense are used, as seems to be the case, one is impaled on an equally unpleasant horn of the dilemma. This horn concerns irrational, transcendental and imaginary numbers. These are increasingly implausibly empirically explainable. In the case of the irrational numbers (those of which are neither transcendental nor complex as well, that is), one could argue that one has a potential irrational number - “in the limit of infinite measurements” (of, say, the diagonal of a square of side 1), or something along those lines. For this to be satisfactory, one must investigate Armstrong’s account of possibility. This occurs in chapter 10 of his (1997). Here he must find “truthmakers” for possibilia. He suggests (page 150) that truthmaker(s) for a particular modal truth work just in virtue of relations of strict identity and difference holding between the constituents of the truthmaker. We are not given much here to determine what the possibility in the above context is to mean. It appears, based in particular on remarks on page 169, that there <u>could</u> be a complicated composite state of affairs in which the above conditions (i.e. an infinite measurement)[7] are realized (in an infinite universe of the appropriate sort). This doesn’t seem too satisfactory, and I think perhaps that I have missed something vital in this section, and will dwell on it little.

But this kind of complication becomes worse, as to get

[7] It is not clear whether the potential measurement (conceptually or otherwise) ‘is’ the number or the magnitude is. Armstrong’s quote from Newton (1997, pg. 179) suggests the former. Nothing much hinges on this distinction, however.

transcendental numbers[8], one cannot specify a single ‘ratio operation’ by <u>which</u> they are to be obtained. Yes, it is true that (e.g.):

$$\pi = \sqrt{6(\sum_{n=1}^{\infty} \frac{1}{n^2})}$$

but, it is not clear how one could encounter that in the world in any fashion. How does it <u>ontologically relate</u> to the π we “constructed” (or abstracted) from circles? Armstrong’s situation is even more complicated when it comes to, say[9]:

$$\ln 2 = \sum_{1}^{\infty} \frac{(-1)^{n-1}}{n}$$

The gist of the above is that it is possible to <u>imagine</u> (in infinite time) finding an infinite conjunction representing the above infinite sums. (No finite number of sums would ever give you give the transcendental number). As for complex (and imaginary numbers), the present author is completely baffled as to how to “operationalize” them. Limitations of one’s imagination do not show what is possible, but here the burden of proof is clearly on Armstrong to show us this, at the very least.

I would argue that one should cut off the hierarchy of operationalized numbers right at the beginning. On a fictionalist account numbers aren’t ontologically real - they are not things, nor objective properties or relations (Douglas 2000). Only

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[8] To be strictly correct, this applies to irrationals like $\sqrt{2}$ as well, but we shall Armstrong to make use of ratios as long as possible. Conceivably he could say the aggregate of the 1-1-$\sqrt{2}$ triangles embodies the $\sqrt{2}$: 1 ratio he wants. My discussion of logarithms (below) considerably complicates this discussion, as it has no geometric analogue.

[9] Adopting Newton’s view might be the problem here. The first of these infinite sums was not worked out until after Newton’s death (Dunham 1990). It seems like it could be anachronistic to adopt his views on numbers in this light.

properties which come in degrees are, and sometimes those degrees are not representable by natural numbers[10]. Matters of concreteness only get worse with more complicated mathematical objects, such as groups.

As noted in Baumslag & Chandler (1968), a group is an ordered pair (S,•) where S is some set and • is some binary operation that satisfies three postulates. How does one go about placing that in the context of sets and classes according to Armstrong? Once again we have to ask whether to what extend he intends to 'reduce'. There's a possibility that groups are a really complicated (potential or actual) complex state of affairs of state of affairs of (etc.), but that leaves the binary operation somewhat ill defined. Is it to be regarded extensionally? Or as one of the physical relations that Armstrong admits in the world?

We have met already one of Armstrong's physical relations, namely conjunction. Are there any others? In particular, we have one of the standard boolean operations defined "operationally", but the all important negation is not defined so we cannot find complements of classes. These complements would be relative to some larger class, perhaps the aggregate of states of affairs that makes up the world[11] . Here a brief look at Armstrong's account of logic might be useful, as a possibility (albeit a strange one) is that he might regard all boolean algebras as "really" referring to the same sorts of things. We have met Armstrong's account of logic briefly before when we discussed "partial identity". Here, let us examine it *de nouveau* in the light of the present concern.

On page 183 of *A World of States of Affairs* (Armstrong 1997) we are given Armstrong's brief remarks on logic, in the context of logical and mathematical truths. The details there are not terribly complete. We are told that if the account of mathematical

[10] This remark should not be taken historically. The issue of how numbers arose historically is a different matter. Natural numbers may well have been suggested by things which were to be counted, but that is rather different from saying those numbers are ontologically connected in some way to that which is counted.

[11] Armstrong's 'class of all classes' thus would be a class, but not contain itself. It would, however, have itself as an improper part.

truth in the preceding pages of this chapter is successful, a similar account can be told about logical truths. As far as I can tell, this does not tell us anything about concretizing the rest of either the set theoretic operations or those from a general boolean algebra. He does tell us that “forall” can be handled by the class concertizing material in the following chapter. Since one ‘obvious’ way to concretize “forall” would involve a repetition of conjunction operations this does not give us any further hints on the lines of other aspects of logic.

However, I have one half-baked suggestion for negation in Armstrong’s lines, as follows. Given a composite state of affairs a+b+c, say, ~b is simply the state of affairs without b, so in the state of affairs a+b+c, ~b = a+c. But using this and an analogue to De Morgan’s laws to produce a definition of an ‘or’ type operation does not work in Armstrong’s system. Here I think this failure to easily generate other operations in a ‘natural’ way is indication that Armstrong’s conjunction operation is ill specified. It seems to refer to both “conjunctive states of affairs” and “conjunction of states of affairs”, with no way of distinguishing the two, and that creates the problem. (It is also used in a concrete and a semi-conceptual fashion, as well.) It should not be said that all possibilities have been excluded - perhaps a less ‘natural’ account would work well with Armstrong’s account.

Another place where the metaphysics of Armstrong’s account makes his philosophy of mathematics concerns Rosen’s objection, noted above. This again takes us back to the notions of possibility at work in Armstrong’s account. Does Armstrong’s metaphysics commit us to equating mathematical possibility with ontological or nomological possibility? Rosen’s paradox cannot be simply stipulated away by assuming that there in principle can be a higher order state of affairs in which the class under consideration is embedded. Armstrong must say that we abstract away from what is the case at some point. (He seems to already think that numbers are like this, to some degree, as the only other alternative would be to say that ‘three’ <u>is</u> the mereological union of the aggregates ***, !!!, and many others, and that seems implausible.)

I mentioned above that Armstrong’s mereology cum class theory is a bit unclear on the relations between set inclusion and set membership. Mathematically it is difficult to work with giving everything a name and, if the (intuitionistic-like) requirement is added that we must be able to actually pick out members to form a subclass, problems may arise with segments of the real line, and so forth. Take verifying the expression $\{x \in \Re : x \in [-1,1]) \subset \{y \in \Re : y \in [-10,10]\}$, for instance. If one has to imagine checking each and every one of the nondenumberable number of ‘little bits’ in the first set against the second, the problem of understanding this arises. (With notions of continuity, perhaps one could save Armstrong, but it seems odd to have to bring such sophisticated notions in to solve a set theoretic problem. This relates back to earlier worries about set theory as foundations of mathematics noted above.)

We have already remarked that Armstrong and Maddy have similar concepts of class and set. Can we apply Maddy’s (1990) axioms in an attempt to save Armstrong on any of the above problems?

Maddy’s axioms are mostly a standard set of ZFC axioms, with the two following modifications. One, her axiom of extentionality looks like this, where x, y and z range over sets:

$$\forall x \forall y \forall z((z \in x \leftrightarrow z \in y) \rightarrow x = y)$$

Note that this identifies individuals with singletons. In order to complete the new set theory, Maddy needs to modify the usual Axiom of Foundation and add two new axioms about individuals. Foundation gets modified because each individual is now self membered. It now reads “each non-empty set contains either an individual or an epsilon-least element” (Maddy 1990, pg. 285).

Thus we (secondly) turn to the final piece of this new set theory, the two new axioms about individuals. These are:

$$\exists x \exists y(x = \{x\} \ \& \ y = \{y\} \ \& \ x \neq y)$$

$$\exists x \forall y(y \in y \rightarrow y \in x)$$

We need an axiom guaranteeing at least two unique individuals, says Maddy, because without the postulate we cannot construct ordinal-like sets.

If we use Maddy's set theory as a more general version of Armstrong's class theory cum mereology, we can partially solve the worries about boolean operations, as one could regard them as fictitious elements in a factual theory. This is much like the null individual is in some other mereologies. It is not clear whether Armstrong would accept this "ideal entity" in that sense. It is also unclear how properties of this set theory, such as De Morgan's Laws (mentioned above), are to be interpreted in this light. Furthermore, under Maddy's scheme, there are at least two unique individuals from which the rest of the objects of the theory can be constructed. If Armstrong were to make use of her set theory, he ought to give a hypothesis of what these individuals are. I have no suggestions on these least bits of the world - perhaps Higgs particles (see Lederman 1993) would do. It is also not clear whether the postulate that there must be no less than two of them has any ontological significance. Finally, an object on this Armstrong-Maddy account would occasionally be identical with it own embedding in a state of affairs. This suggests the (fruitful) metaphysical hypothesis that there are no propertyless things. Further investigation in these latter directions would be needed.

Worries over logic are also solved by Maddy's set theory, but probably not in a way Armstrong would like. It appears that she holds that logic is strictly nonempirical (unlike set theory). In that respect she is more in line with the practice of most working mathematicians and logicians. However, it is not clear that Armstrong would find this solution acceptable. As remarked earlier, he feels that logical truth is 'in the world' in a similar fashion as mathematical truth. Thus at best he could take Maddy's (implicit) logic as a fallible approximation to the 'true logic'. This sounds curiously like Kurt Gödel's position as reported in Wang (1995 [1989]).

It seems that Maddy’s account also solves the worry about inclusion and membership, as it does give a rigorously specified way in which inclusion is a short hand. This would allow Armstrong to regain some of the foundation views he was close to discarding, above (for instance, that $2 \subset 3$).

Section 3: Remarks on Metaphysics and Mathematics

As intimated throughout, I regard some of the problems with Armstrong’s account as originating in his apparently running roughshod over the subtleties of the conjunction operation he has discussed. As already noted it seems to represent both a formal operation and an ontological one. Now, Armstrong could have it that the formal operation he has “abstracted” from cases in the real world is simply in error. This would be (in general) an acceptable solution, but it would mean overcoming the ambiguity between two readings of conjunction of universals I have outlined above. It would also require that some account of negation be given. Here I must return to remarks about fiction in factual theories. As remarked, there are many factual theories in which various kinds of fiction occur; these can be objects with idealized referents (e.g. bodies isolated from all forces in Newtonian mechanics). They may also be conceptual items with no existence at all (hamilitonians come to mind). Can Armstrong make use of a fictitious negation operation, or must he concretize it too? I suggested above that in order for De Morgan’s Laws to work the latter likely must be the case. One can defend this thesis further on grounds of semantic homogeneity, which is a general worry one could have about attempts to bring mathematics into metaphysics. To be fair, it also infects other branches of mathematical realism (Platonism, likely including a Leibnizian idiosyncratic version of it), not just Armstrong’s.

The other general concern I have is the danger of simplicism. This concerns whether one overarching principle can account for everything there is. In this case, the notion of “state of affairs” is being used to elucidate ‘everything under the sun’. This worry affects both the mathematical and the metaphysical worries, and in some sense, both together. This is, I think, because Armstrong has attempted to build a philosophy of mathematics into his metaphysics directly. There is a past history of trying to do this (e.g. Plato, Gödel) and these are generally regarded as failures, at least amongst many philosophers. It is instructive to note that Armstrong recognizes the excesses of (particularly) the Platonistic conception of universals when it comes to nonmathematical ones. Armstrong called this metaphysics Aristotlian, which seems as good a name as any. But his philosophy of mathematics is not so much Aristotlian as it is Kantian and Lockean.

Both Kant (1996 [1787])[12] and Locke (1975 [1700]) tried to concretize mathematics in very similar ways to Armstrong, and their philosophies of mathematics apparently did not survive. This is a strong indication (though admittedly a fallible one) that they did not match practice or content of mathematics.

Another reason to divorce mathematics from the world itself is to account for its portability across disciplines. One and the same equation can model numerous different phenomena. It is not immediately how this is to be the case if the “building blocks” of the mathematics doing the modeling actually refer to something. Again Armstrong can escape from this by saying that the collection corresponding to each building block IS the mathematical feature, but then one falls into his account of classes as mereological wholes again, and back into explaining why parts of wholes should explain ‘everything’. This is related to the simplicism worry above. If pushed in another direction, towards the “idealization of real things” dimension, it is not clear where to stop this. (See above in the discussion of numbers.) Hence it is not clear that one should not return to a purely conceptual view of mathematics (weak fictionalism, formalism, some varieties of structuralism).

It is also difficult to see that theories that are formally similar (e.g. electrokinetics and the kinetic theory of heat) are <u>factually</u> similar enough in some respect to warrant saying their

[12] For instance, Kant at A 163 / B 203 writes why extensiveness (and hence magnitude in general) involves <u>space</u>. Note that other passages use magnitude in reference to numbers (particularly at A 164-165 / B 205-206).

formal features are ontologically connected somehow. Divorcing mathematics from the world escapes this problem (as in a fictionalist philosophy of mathematics), as on this account mathematics itself does not refer to anything in particular at all.

Conclusion

I have analyzed two strains of thought (general metaphysics and philosophy of mathematics) in Armstrong’s 1997 work that strike me as being a little weak. Each of them suffers from interrelated problems of which I have hopefully diagnosed correctly and hinted at directions for further investigation to solve and move beyond them.

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